

Reference Sheet 2.7.
Rules of Propositional Logic

I. BASIC ARGUMENT FORMS

M.P. $\frac{p \supset q}{p}$ $\therefore q$	M.T. $\frac{p \supset q}{\sim q}$ $\therefore \sim p$	Hyp. Syll. $\frac{p \supset q}{q \supset r}$ $\therefore p \supset r$	Dilem. <table style="width: 100%; border: none;"> <tr> <td style="border: none;">$p \vee q$</td> <td style="border: none;">$p \vee q$</td> </tr> <tr> <td style="border: none;">$p \supset r$</td> <td style="border: none;">$p \supset r$</td> </tr> <tr> <td style="border: none;">$q \supset r$</td> <td style="border: none;">$q \supset s$</td> </tr> <tr> <td style="border: none;"><hr style="width: 50%; margin-left: 0;"/></td> <td style="border: none;"><hr style="width: 50%; margin-left: 0;"/></td> </tr> <tr> <td style="border: none;">$\therefore r$</td> <td style="border: none;">$\therefore r \vee s$</td> </tr> </table>		$p \vee q$	$p \vee q$	$p \supset r$	$p \supset r$	$q \supset r$	$q \supset s$	<hr style="width: 50%; margin-left: 0;"/>	<hr style="width: 50%; margin-left: 0;"/>	$\therefore r$	$\therefore r \vee s$										
$p \vee q$	$p \vee q$																							
$p \supset r$	$p \supset r$																							
$q \supset r$	$q \supset s$																							
<hr style="width: 50%; margin-left: 0;"/>	<hr style="width: 50%; margin-left: 0;"/>																							
$\therefore r$	$\therefore r \vee s$																							
Simp. $\frac{p \& q}{p}$ $\therefore p$ $\therefore q$	Conj. $\frac{p}{q}$ $\therefore p \& q$	Disj. Syll. $\frac{p \vee q}{\sim p}$ $\therefore q$	Disj. Add. $\frac{p}{p \vee q}$ $\therefore p \vee q$ $\therefore q \vee p$																					
D.Thens <table style="width: 100%; border: none;"> <tr> <td style="border: none;">$p \supset (q \& r)$</td> <td style="border: none;">$p \supset q$</td> </tr> <tr> <td style="border: none;"><hr style="width: 50%; margin-left: 0;"/></td> <td style="border: none;">$p \supset r$</td> </tr> <tr> <td style="border: none;">$\therefore p \supset q$</td> <td style="border: none;">$\therefore p \supset (q \& r)$</td> </tr> <tr> <td style="border: none;">$\therefore p \supset r$</td> <td></td> </tr> </table>		$p \supset (q \& r)$	$p \supset q$	<hr style="width: 50%; margin-left: 0;"/>	$p \supset r$	$\therefore p \supset q$	$\therefore p \supset (q \& r)$	$\therefore p \supset r$		Bicond. <table style="width: 100%; border: none;"> <tr> <td style="border: none;">$p \equiv q$</td> <td style="border: none;">$p \supset q$</td> </tr> <tr> <td style="border: none;"><hr style="width: 50%; margin-left: 0;"/></td> <td style="border: none;">$q \supset p$</td> </tr> <tr> <td style="border: none;">$\therefore p \supset q$</td> <td style="border: none;">$\therefore p \equiv q$</td> </tr> <tr> <td style="border: none;">$\therefore q \supset p$</td> <td></td> </tr> </table>		$p \equiv q$	$p \supset q$	<hr style="width: 50%; margin-left: 0;"/>	$q \supset p$	$\therefore p \supset q$	$\therefore p \equiv q$	$\therefore q \supset p$		R.A.A. <table style="width: 100%; border: none;"> <tr> <td style="border: none;">$p \supset q$</td> </tr> <tr> <td style="border: none;">$p \supset \sim q$</td> </tr> <tr> <td style="border: none;"><hr style="width: 50%; margin-left: 0;"/></td> </tr> <tr> <td style="border: none;">$\therefore \sim p$</td> </tr> </table>	$p \supset q$	$p \supset \sim q$	<hr style="width: 50%; margin-left: 0;"/>	$\therefore \sim p$
$p \supset (q \& r)$	$p \supset q$																							
<hr style="width: 50%; margin-left: 0;"/>	$p \supset r$																							
$\therefore p \supset q$	$\therefore p \supset (q \& r)$																							
$\therefore p \supset r$																								
$p \equiv q$	$p \supset q$																							
<hr style="width: 50%; margin-left: 0;"/>	$q \supset p$																							
$\therefore p \supset q$	$\therefore p \equiv q$																							
$\therefore q \supset p$																								
$p \supset q$																								
$p \supset \sim q$																								
<hr style="width: 50%; margin-left: 0;"/>																								
$\therefore \sim p$																								

II. BASIC EQUIVALENCE FORMS

D.Neg.	$p = \sim(\sim p)$
DeMorg.	$\sim(p \& q) = \sim p \vee \sim q$
DeMorg.	$\sim(p \vee q) = \sim p \& \sim q$
Contrap.	$p \supset q = \sim q \supset \sim p$
Cond.	$p \supset q = \sim p \vee q$
Bicond.	$p \equiv q = (p \supset q) \& (q \supset p)$
D.Ifs	$(p \& q) \supset r = p \supset (q \supset r)$
Dupl.	$p \& p = p$
Dupl.	$p \vee p = p$
Comm.	$p \& q = q \& p$
Comm.	$p \vee q = q \vee p$
Assoc.	$(p \& q) \& r = p \& (q \& r)$
Assoc.	$(p \vee q) \vee r = p \vee (q \vee r)$
Dist.	$p \& (q \vee r) = (p \& q) \vee (p \& r)$
Dist.	$p \vee (q \& r) = (p \vee q) \& (p \vee r)$

III. Logical Truths

(Taut.)
• $p \vee \sim p$
• $\sim(p \& \sim p)$
• $p \supset p$
• $(p \& q) \supset p$
• $(p \& q) \supset q$
• $p \supset (p \vee q)$
• $p \supset (q \vee p)$

IV. Assumption Rules (§ 2.10)

<table style="border: none; width: 100%;"> <tr> <td style="border: 1px solid black; padding: 2px;">p</td> <td style="padding: 2px;">assmp</td> </tr> <tr> <td style="border: none; padding: 2px;">:</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">q</td> <td style="padding: 2px;"></td> </tr> </table> $\therefore p \supset q$ <p style="text-align: right;">C.P.</p>	p	assmp	:		q		<table style="border: none; width: 100%;"> <tr> <td style="border: 1px solid black; padding: 2px;">p</td> <td style="padding: 2px;">assmp</td> </tr> <tr> <td style="border: none; padding: 2px;">:</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="border: 1px solid black; padding: 2px;">$q \& \sim q$</td> <td style="padding: 2px;"></td> </tr> </table> $\therefore \sim p$ <p style="text-align: right;">I.P.</p>	p	assmp	:		$q \& \sim q$	
p	assmp												
:													
q													
p	assmp												
:													
$q \& \sim q$													