

## Reference Sheet 4.4. Rules of Quantificational Logic

In what follows,  $\alpha/\beta$  indicates putting  $\alpha$  for all occurrences of  $\beta$ , and  $\alpha//\beta$  indicates putting  $\alpha$  for some occurrences of  $\beta$ .

### The Quantifier-Negation Laws

- |          |   |
|----------|---|
| Q.N.     | $\sim(\forall x) Fx = (\exists x) \sim Fx$                    |
| Q.N.     | $\sim(\exists x) Fx = (\forall x) \sim Fx$                    |
| Cat.Q.N. | $\sim(\forall x)(Fx \supset Gx) = (\exists x)(Fx \& \sim Gx)$ |
| Cat.Q.N. | $\sim(\exists x)(Fx \& Gx) = (\forall x)(Fx \supset \sim Gx)$ |

### Universal Instantiation U.I.

- $$\frac{(\forall x)(\dots x \dots)}{\therefore (\dots n/x \dots)}$$
- No restrictions on the name **n**.

### Existential Instantiation E.I.

- $$\frac{(\exists x)(\dots x \dots)}{\therefore \text{select name } n}$$
1. **n** is a name that has never been used before  
 2. **n** must first be introduced in a selection step

### Existential Generalization E.G.

- $$\frac{(\dots n \dots)}{\therefore (\exists x)(\dots x/n \dots)}$$
- $$\frac{(\dots n \dots)}{\therefore (\exists x)(\dots x//n \dots)}$$
- No restrictions on the name **n**.

### Universal Generalization U.G.

- $$\vdots$$
- select name **n**

$$\vdots$$

$$(\dots n \dots)$$
1. The first line selects a name **n** never used before.  
 2. The last line is not un-representative.
- $$\therefore (\forall x)(\dots x/n \dots)$$